Learning mathematics through activities with robots

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Abstract

There are several countries that integrate programming into their mathematics curricula, thereby making robotics an interesting aspect of mathematics education. However, the benefits of using robotics for mathematics education are still unclear. This article addresses the use of mathematical tools with robot-based problem-solving activities by discussing how mathematical tools are used in robot-based activities. This ethnographic intervention study took place in one secondary school in Norway as a part of an elective class in which videotaped data were gathered by observing the activities of a group of two or three students using Lego Mindstorm robots during an eight-week period. Through the use of activity system analysis in Cultural Historical Activity Theory, the analysis found that students use different kinds of mathematical tools. Furthermore, mathematics can change its role from instrumental tool to object, that is, to an integrated aspect of the purpose of the activity.

Keywords: Robots, mathematics education, cultural historical activity theory, activity system analysis, mathematical tools, object of activity

Introduction

Education systems in various countries are integrating the teaching of programming into their curricula in a variety of ways, by including general information and communications technology courses and by integrating programming into individual subjects. Nordic countries such as Finland, Sweden, and Norway have integrated or are planning to integrate programming into the mathematics curriculum. A pedagogical discussion regarding the merits of integrating programming in a cross-curricular approach (Balanskat & Engelhardt, 2015) is necessary. There is a need for research-based knowledge on issues such as how programming can be linked with differing subject areas, how programming influences students' learning, and the interplay between pedagogical approaches to different kinds of programming with different kinds of tools and assessments (Balanskat & Engelhardt, 2015; Bocconi, Chioccariello, & Earp, 2018).

Currently, there are dozens of different robots and toolkits suitable for educational use (Karim, Lemaignan, & Mondada, 2015), with Lego Mindstorm robots being the most widely studied (Benitti & Spolaôr, 2017). In classrooms, students can steer and control Lego Mindstorm robots by programming motors with the help of a variety of pieces, sensors, and blocks (Savard & Freiman, 2016). How curriculum-related mathematics in robot-based activities is used is unclear. Savard and Highfield (2015) argued that even teachers cannot associate the mathematics used by students in robot-based activities with curriculum-related mathematics. According to Savard and Freiman (2016), students do not design the use of mathematical tools during the problem-solving activities with robots but concentrate instead on digital design.

The aim of this article is to contribute to pedagogical discussions regarding programming and robotics in mathematics education by taking a closer look at the use of mathematical tools in students' collective activities with robots. We achieve this by analyzing students' activities with Lego Mindstorm robots, drawing on Engeströms' (1987) Cultural Historical Activity Theory (CHAT), which is well suited for analyses of tool-mediated collective activities. In the perspective of CHAT, the use of tools is dependent on the object of the activity. The component of the object has a special and central role in CHAT. The object of the activity is understood in CHAT as a goal, motive, drive, direction or purpose, which subjects of activity aim collectively. With robots the object of the activity could for instance be, to program the robot to drive a certain path.

A more detailed use of mathematics in students' collective activities with robots is discussed by answering the following question: What is the relationship between mathematical tools and objects in robot-based collective student learning activities in secondary education?

In our earlier article, we have addressed the role of the teacher in students learning processes with robots. We discussed the relationship between role of the teacher and other components in students' activity development (Forsström, 2019). We found out that the role of the teacher in the beginning of the activity development influences the object (drive, direction and purpose) of activity and mathematical tools in use. However, the article did not discuss the activity development in mathematical tools mediated activity. Thus, this article concentrates on the relationship between mathematical tools and object (drive, direction and purpose) of the activity during activity development.

We want to analyze how the use of mathematical tools develops in situations in which students work in collaboration on a relatively open-ended task. In these situations, students are not obligated to use mathematics but might find it useful in the process of their activities and tasks. Furthermore, in this article, our interest is not individual, cognitive learning but learning processes in collective interaction and activity. In particular, we want to analyze how a group uses tools and how the group negotiates the object of the activity, that is, the purpose and motivation of their project. Learning is seen as a collective, transformative, and expansive process (Engeström, 1987; Kaptelinin & Nardi, 2006).

The robot activity of one group of students, aged 12–13, took place during an eight-week period, and was chosen as the unit of analysis. The data material was gathered through video recording and field notes based on observation. The evolving interaction between the human actors, robot, and mathematical tools was at the centre of attention. A micro-strategy allowed a detailed analysis of how mathematical tools are used in different manners and how the drive, purpose, motivation, and direction of the activity can be developed and changed.

Following this introduction is a review of the central literature discussing learning opportunities through activities with robots. A section on the theoretical framework of this study, CHAT, is provided next. In the methodology section, the research strategy, sampling constitution of data, and strategies of analysis are discussed. The principal section is partly the analysis of the use of tools and partly object development and expansion. The final section discusses how these findings contribute to existing literature.

Literature review

It is unclear how robots serve the curriculum in practice (Alimisis, 2013). Review articles, such as Alimisis (2013), Benitti and Spolaôr (2017), and Karim (2015), discussing the educational benefits of robotics, in general, have revealed a need for studies discussing the curriculum connection with robotics. Several studies considering robots in education have been conducted as part of an out-of-school activity or as an extracurricular activity (Benitti & Spolaôr, 2017).

Problem-solving activities with robots offer a different kind of learning environment in mathematics education by providing the opportunity to use mathematics in practice (Ardito, Mosley, & Scollins, 2014; Barak & Assal, 2018). The educational benefits of robotics in mathematics education are still unclear. Quantitatively, Lindh and Holgersson (2007) found that some groups of students improved their results in mathematics tests after training with Lego Mindstorm robots but with some of the groups, no improvement was noticed. The post-test results were compared with the pre-test results.

Qualitatively, Barak and Assal (2018) argued that, even if robotics can provide an informal and innovative learning environment and enrich mathematics learning by providing mathematics in action, it cannot substitute for systematic and formal mathematics teaching because of its informal nature. However, Bartolini Bussi and Baccaglini-Frank (2015) found out that the first grade students connected their informal activities with bee-bot-robots (programmable toy that resembles a bee) with formal mathematics concept of square. The students programmed the robot to drive an O-letter path. Because the robot turns only 90 degrees at time, the students called the path to “squarized O”, which consists of four right angles. This is an example how young students connected a formal mathematics concept in the informal activities with robots. Bartollini Bussi and Baccaglini-Frank (2015) argued that activities with robots might have the potential to open also other formal mathematical meanings for the students.

Savard and Freiman (2016) found that students used mostly a trial-and-error strategy in problem-solving activities with robots. Students often started with digital contexts without creating any design regarding the use of mathematics; mathematical tools were mostly in use through the trial-and-error strategy. Savard and Freiman (2016) argued that trial and error worked well in solving programming problems with robots, but it also acted as an obstacle to students in acquiring greater understanding in mathematics because students could not detect a source of error that they made within the mathematical context.

Large, quantitative studies, such as Lindh and Holgersson (2007), understand and assess learning as an individual change in a subject's knowledge. Students might solve problems in groups, but the tests and grades are individual. By contrast, Savard and Freiman (2016) used a sociocultural approach to gain understanding in the emerging mathematical reasoning. They conducted in-depth investigation regarding students' learning processes to acquire a better understanding of the complexity of assessing students' learning in mathematics through the activities with robots. On the issue of learning, they argued, "Knowing that students successfully performed the task is not enough: knowing which concepts and processes they used gives more information to position them within their learning process" (Savard and Freiman 2016, p. 109).

This means that investigating the effects of specific methodological approaches in education is not sufficient. Understanding the complexities and processes of learning as participation in collective activities is vital. In this article, we argue that different uses of mathematical tools and different objects in the activity give rise to very different learning processes and possibilities.

Theoretical framework

To analyze the relationship between mathematical tools in use and objects in robot-based activities, we examine students' learning processes with robots by drawing on CHAT, an analytical framework offering a reservoir of concepts, possible relations, and processes. The framework as a whole understands human action as social activities, and the analytical reservoir enables the analyses of the different aspects of and processes in and between activities. In this paper, we analyze the interaction of the group as an activity. The group consists of the students, the teacher, and the robot.

CHAT enables the analysis of interaction, that is, the interactive processes in the group activity. Furthermore, CHAT assists in understanding learning and change in the group as mediated by tools (Engeström, 2005). Having access to constructive tools and knowing how to use them are important in learning processes. Furthermore, any activity is constantly changing, developing, and shaping itself, and the activity system analysis in CHAT enables seeing the effects of various components, such as tools and objects, for that development (Engeström, 1987).

In Engeström's (1987) activity system analysis (Figure 1), seven components, namely, subject, object, tool, rules, community, division of labor, and outcome, are connected. The components are listed in table 1.

Fig. 1 Activity system developed by Engeström (1987, p.78).

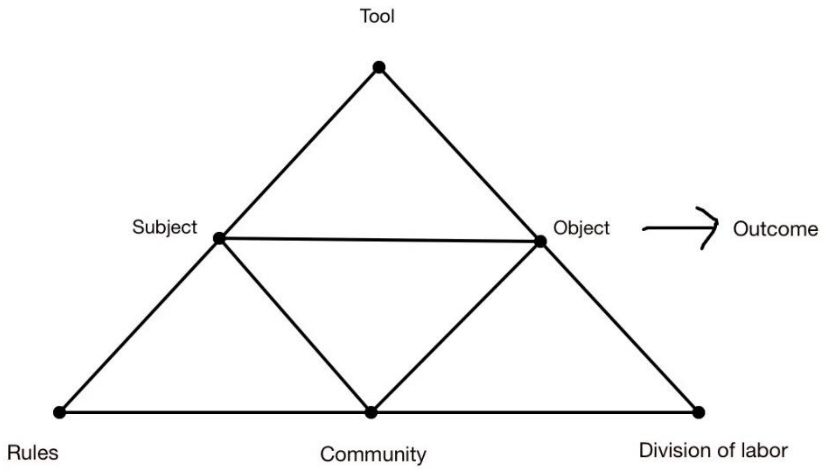


Table 1 Definitions of the different components in the activity system analysis

Component	Definition/meaning	Examples from this study
Subject	Individual or group of people engaging in the activity (Yamagata-Lynch, 2010)	Acting students and teacher
Object	Driving force in the activity (motive and goal) (Engeström, 1987)	Fulfill a task with the robot
Tool	Instrument mediating the activity (Engeström, 1987)	Robot, computer, mathematical tools, programming (coding)
Rules	Regulations relevant to the activity (Yamagata-Lynch, 2010)	Task assignment and rules from the mathematics classroom
Community	Social group the subject belongs to during the activity (Yamagata-Lynch, 2010)	Entire class of students and teacher
Division of labor	How the tasks are shared during the activity (Yamagata-Lynch, 2010)	Collaboration between students and the role of the teacher
Outcome	Result of the activity (Yamagata-Lynch, 2010)	Robot drives a track as programmed

Object of the activity

In the activity system analysis, activities are motivated and led by objects, an activity is always object-orientated (Engeström, 1987). In this context, the object determines the activity, and the activity is recognized and distinguished from other activities by its object. The concept of object has a special definition in CHAT as a collective goal, motive, direction or driving force in the activity (Engeström, 1987; Roth & Radford, 2011). Thus, the definition of object in CHAT differs from traditional everyday understanding about word object as a material thing or item.

Through a division of labour, subjects in the activity work collectively towards the same object (Engeström, 1987). The participants can manipulate and transform the shared objects, which can be a material or nonmaterial such as a plan or common idea. The object can also be changed during the activity (Kuutti, 1996). The subject aims towards the objects through tools (Engeström, 1987).

Tools

The subject relates to the object through the use of various tools. The use of tools depends on the objects of the activity (Engeström, 1987). The tools of an activity can be a material, such as computers and robots, or nonmaterial, such as rules, recipes, stories, and narratives. A language can be seen as a tool that enables communication. Mathematics is also a language (Ryan & Williams, 2007). A programming language enables communication with robots.

The activity is always tool-mediated and collective. Even though it appears that an individual has a direct contact with the object, there is always a connection with other individuals at least through some cultural tools such as gestures, pictures, or words. As activities are always collective, tools are also the results of the collective activities. The cultural tools, which enable collective activities related to other individuals, are the results of human beings’ collective life activities in practice (Engeström, 1987). Mathematics is a cultural tool, created over time by human beings. Different kind of mathematical tools can be, for instance, different formulas, algorithms, proportions, functions, and graphical models.

Often the use of tools is unconscious (Engeström, 1987). The focus can temporarily be on a tool, for example, when robots do not act as desired, and the students focus on the robots. However, this can only be a temporary state. Tools are not objects of the activity (Engeström, 1987).

Expansive learning

Unlike traditional learning theories, CHAT links learning with social transformations by linking individuals with social structures (Engeström, 2005). Learning is seen more as a long-lasting collective and expansive process than an individual result. The focus is on object development:

> Traditionally we expect that learning is manifested as changes in the subject, i.e., in the behavior and cognition of the learners. Expansive learning is manifested primarily as changes in the object of the collective activity. In successful expansive learning, this eventually leads to a qualitative transformation of all components of the activity system. (Engeström & Sannino, 2010, p. 8)

Traditional learning theories see an individual as a separate acting subject and learning as a process in which individuals acquire stable knowledge that can be identified with changes in the subjects' behaviors. In this kind of situation, the teacher knows in advance what students are to learn (Engeström, 2005). Learning cannot be predicted in advance in problem-solving activities with robots because the learning process depends on the students' collective and individual choices during the activities. For example, the teacher cannot predict what type of mathematical tools her students are going to use when solving problems.

In expansive learning, owing to the transformative processes in the activity, the change in the object provides wider learning possibilities. The changes in the object constructed by the learners provide opportunities for them to learn "something that is not yet there" (Engeström & Sannino, 2010, p. 2). According to Engeström (2005, p. 64) "[a]n expansive transformation is accomplished when the object and motive of the activity are re-conceptualized to embrace a radically wider horizon of possibilities than in the previous mode of activity."

During the development of activities, tensions might arise in or between different components in the activity system or between different activity systems. These tensions often change the activity in an innovative manner and create the possibility of expansive transformations. Some participants might question and redirect the activity as a result of contradictions and tensions. That can cause deliberate collective efforts towards change in the activity. A change in the object with several possibilities causes expansive transformations. This kind of collective and transformative process is a part of expansive learning (Engeström, 2005).

Engeström in his later works focused on expansive learning among several activity systems and paid less attention to separate activities. We analyze one activity, the group activity in the classroom, and therefore use primarily Engeström's earlier work.

Research methods

Research context and design

This study was conducted in one primary school in Norway. Norwegian schools are interesting because programming is becoming a part of the mathematics curriculum in Norway, and its school system has a positive attitude towards technology (Utdanningsdirektoratet, 2013, 2018). The school that we chose is of interest for this study because it represents a regular Norwegian medium-sized lower secondary school. The cooperation was natural because one mathematics teacher in that school was about to integrate robots into his teaching in an elective class called "Technology in Practice."

The compulsory Norwegian school consists of a 10-year elementary school. The education is based on the national curriculum in which mathematics has a central role. Mathematics is seen as a part of cultural heritage and the basis of logical thinking. Problem solving is seen as an important component in mathematical competence. In any event, although programming is not yet part of the curriculum in Norway, technology is still strongly present. The use of technology is recommended in most of the mathematical activities (Utdanningsdirektoratet, 2013).

A variety of research strategies were discussed. We wanted to study everyday educational practice, since it is not our aim to analyze or test a best case. Furthermore, we needed a strategy that will enable us to follow the robot activity in great detail in terms of action, interaction, conversation, arguing, tensions and conflicts, interaction with the robots, and use of different bits and pieces of mathematics. To analyze processes and development, we needed to follow the same students over a period of time. We decided to follow one class for one semester, with a combination of observation and video recording of all sessions. Investigating social practices in natural settings is a characteristic of ethnography, understood as in Madden (2017, p. 16): "Ethnography is a qualitative social science practice that seeks to understand human groups (or societies, or cultures, or institutions) by having the researcher in the same social space as the participants in the study."

More specifically, we followed a study design called focused ethnography, which differs from traditional ethnography, for instance, through more time-intensive fieldwork, the role of the researchers, and focused observations with key informants (Skårås, 2018). In classroom studies, which followed the design of focused ethnography, it is natural to use videotaping as a data gathering method because the videotaped data material suits well the study of complex processes of learning and teaching (Skårås, 2018). Our study differs from traditional ethnography also with regard to our role as researchers. The teacher conducting the elective class on robots whom we followed lacked any knowledge of Lego Mindstorm. Consequently, we provided him with a short introduction and discussed issues between class sessions. However, the teacher himself planned the class.

Data collection

More specifically, we conducted focused ethnographic fieldwork by videotaping and writing field notes in order to understand the real activity in the classroom when robots entered the scene.

The teacher followed a plan for introducing robots in an elective class for 31 students aged 12–15. The class allowed for more time and space for explorative and creative robot activity than a regular math class would have. Students at ages 12–15 have knowledge both of technology and mathematics. Although, as researchers, we looked for connections with the mathematics curriculum, we did not want to force it. Our interest was in how the students used the mathematics that they had learned. Thus, students and teachers were free to work innovatively without curriculum pressure. Conversely, the fact that the teacher was a mathematics teacher made the connection with mathematics easier.

The assignments designed by the teacher were open; the students were given the opportunity to create their own designs within the tasks, such as what kind of track they programmed the robot to drive on. The open nature of the task enabled a free environment for activity development. The teacher guided the students' activities when it was possible and when they needed to obtain the collective learning. Most of the tasks concerned driving along a particular kind of track with the robots. Some of the tasks were competitive in nature. Students worked in groups of two to four for practical reasons.

The data gathering took place during eight 75-min sessions by observing one group of three students. The students in the group, "Oscar," "Lucas," and "Jacob," were 12–13 years old. This eight-week period was the time required to see the students' entire development from the introduction of the robots to the smooth use of mathematical tools with the robots. The group selection was based on observations and experiments with videotaping during the first sessions. First, it appears that this particular group of students was one of the groups that seemed to enjoy working with robots. Second, their attitude towards the video camera was natural. However, only the last five sessions were videotaped in full because the three first sessions were concerned mostly with building robots and becoming familiar with them. During our systematic observations, the special focus was on changes in the activities, such as changes in objects and tools.

Data analysis

The analysis was divided into three parts. First, the most relevant and interesting video clips from selected sessions that concerned thinking about the use of mathematical tools were transcribed. For this article, we analyzed two sessions in which mathematical tools were in use. The selection of these sessions was based on our observations and field notes. The transcriptions gave detailed accounts of the conversation but not the actions and interactions of the students and the teacher, their bodily and emotional expressions, and the actions of the robot. Therefore, we supplied the transcriptions and our field notes from observation with detailed field narratives based on watching the video clips.

In the second phase of the analysis, we used the whole activity system triangle in CHAT. The transcribed material and our field narratives and notes were coded with the key concepts from the CHAT triangle, namely, tools, subject, object, rules, community, and division of labor. This was done in order to receive a broader view of the activity development.

In the final step of the analysis, in order for the findings and arguments in the article to be pointed out clearly enough, the analysis limited to the relationship between actors, tools and objects. The deeper analysis focused particularly on the use of mathematical tools and the object development in order to answer the research question. As the aim of this study is to discuss the use of mathematics in robot-based activities, the focus was on tools. Furthermore, as the use of different tools depends on the object of the activity (Engeström, 1987), the focus was also on objects of activities. We conducted the deeper analysis by analyzing the relationships between the codes, particularly the relationship between tools and objects, and by analyzing the changes and developments in the codes and code-relations over time. The role of the teacher was obviously important, but also the students' involvement and preparation, their mediation of the response, division of labor, rules and community. The point of the article is not to identify different causal factors throughout the activity, but to interpret it as mathematics having a changed role in the case by focusing on the dynamics between tools and objects.

We determined the object of the activity by identifying the goal or aim, which all subjects of the activity aimed collectively to reach. The tools of activity were identified with the help of the object of the activity. Subjects of the activity needed certain tools to reach their object (Engeström, 1987). The difference between objects of the activities and tools in use was visible by identifying the focus in the activities. The focus of subjects can only temporarily be on tools (Engeström, 2005). For instance, when students are programming the robot, they may need certain mathematical formula or algorithm in order to get a certain value for their program. When students are using that formula or algorithm, the focus is temporarily on mathematical tool. When students obtained the needed result from their calculation, they used it to reach their object, which was to program the robot. The focus was therefore not on mathematics anymore.

Findings

The data of this study were derived from two different sessions. During these sessions, the students attempted to solve a variety of problems, which were partly designed by the teacher and partly by the students themselves. These sessions are briefly presented and then analyzed in more detail.

During Session 1, Oscar was absent, and Lucas and Jacob had difficulties with collaboration. They showed no enthusiasm in working with the robot. Lucas played with the Lego bricks, and Jacob became frustrated with him. Accidentally and by trial and error, they succeeded in programming the robot to drive along a circle with almost the same starting and ending points. At that moment, the teacher was observing the robot’s movements together with the students. On the basis of this observation, the teacher suggested that the students could program the robot to drive along a circle with a radius of 1 m. The students accepted that suggestion and worked with enthusiasm.

In order to program the robot to drive in a circle with a radius of 1 m, the students needed to know how long the robot has to drive and how much it has to turn. The students started solving the problem by determining how much the robot must turn during one wheel rotation and how long the robot must drive using proportions and the circle circumference formula.

The whole turn in EV-3 programming environment is equivalent to the value of 100. With the help of proportion students found out that the value 1 is equal to a 3,6 degrees turn. After that, they found out that the robot has to turn 19.5 degrees during one wheel rotation, because the robot drives 18.5 cm during one wheel rotation and 360:18.5 = 19.5. Furthermore, the students divided 19.5 by 3.6. They used the value 5.5 in their program.

When the students determined the distance that the robot has to drive, they committed an error with the given circle circumference formula, using the radius rather than the diameter and they came up with the answer 3.1415 meters. Thus, the robot drove only half a circle. On that basis, Jacob concluded that they had to double the distance, and the students succeeded with their task.

The students were excited about succeeding in this task, and during Session 2, Lucas and Jacob were willing to apply their learning in a new situation. At the beginning of Session 2, the students were given the new task of driving along a track with the robot, taking hold of a little box, and moving the box along the same track back to the starting point. The students were free to design the track the robot was to drive along by themselves. Lucas and Jacob wanted to have a circle track as a part of the robot's track.

The activity development during these sessions is analyzed in the following section using activity system analysis by focusing on the use of mathematical tools. As the use of tools depends on the object of the activity (e.g., Engeström, 1987), our further analysis concentrated on the object development in the activity.

Based on our analysis, the activity development is divided into four different phases. These phases are discussed and justified in more detail in the following subsections. However, in order to clarify and make it easier to follow our analysis and findings, we present the different phases in the activity development in table 2.

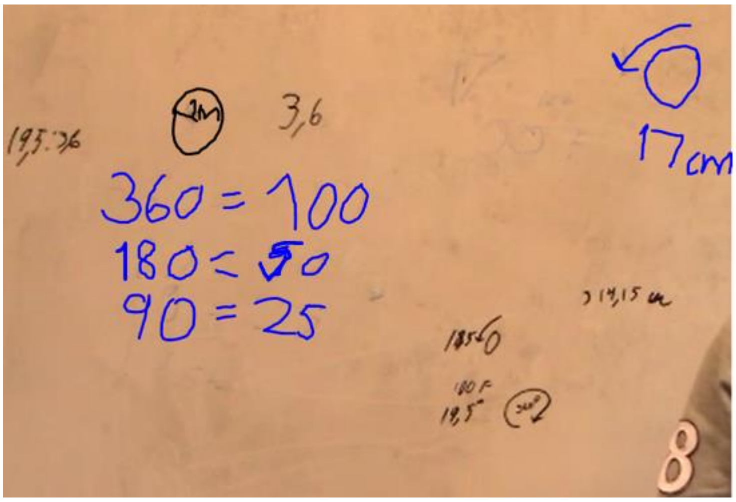
Table 2. The summarization of the components of object of activity and mathematical tools in use during the different phases in the activity development.

	1. The task design	2. The use of mathematical tools	3. Mathematical tool as an object	4. Expansion of the object
Object of the activity	Students started by programming the robot to drive a circle. The teacher mathematized students object by negotiating with students.	The mathematized object, namely to drive the circle with the radius 1 m, enabled the use of mathematical tools.	Because of the error students made with the mathematical tool, mathematics became the object of the activity.	Students wanted to use their learning from last sessions in their new task design. The object of the activity expanded. The new object was to drive a path where a circle track was as a part of the robot's track.
Mathematical tools in use		Students used different types of mathematical tools to reach the object. However, they made an error with the circle perimeter formula.		Mathematical tools were in use again because of the new mathematized object.

Phases 1 and 2: The task design and use of mathematical tools

At the beginning of Session 1, the teacher's suggestion that the students program the robot to drive along a circle with a radius of 1 m motivated the students to collaborate and use mathematical tools. Lucas and Jacob began solving the problem by collaboratively creating their own mathematical tool bank by writing on the whiteboard the mathematical concepts they thought could be useful to them. The students alternated between different roles, with Lucas writing and Jacob suggesting different ideas and vice versa, while they discussed with enthusiasm the kind of mathematical tools they would need to be able to program the robot to drive along a circle with a radius of 1 m. Thus, the teacher's suggestion was the initiator of the students' collective activity, where the driving force, the object of the activity, was to program the robot to drive along a circle with a radius of 1 m. A variety of mathematical tools that the students wrote on the whiteboard mediated the activity. Picture 1 shows a reconstruction of the whiteboard after the students' reasoning.

Picture 1. Reconstruction of what the students wrote on the whiteboard



The teacher’s suggestion to program the robot to drive along a circle with a radius of 1 m was a mathematized version of the activity that Lucas had already begun by programming the robot to drive along a circle using trial and error. The detail of the teacher suggesting the use of a radius of 1 m was pivotal in activating the students to use mathematical tools in their problem-solving activity. The teacher mathematized the students object. Here we understand a mathematized object as an object, which needs to be achieved with mathematical tools. If the object had been only to drive in a circle, without more precisely specifying the size of the circle, the students could have solved the problem by trial and error by changing the values randomly in the program Lucas created at the beginning of the session without planning to use mathematical tools. The trial-and-error strategy had also been seen in earlier studies as an obstacle to using mathematics in problem-solving activities with robots (Savard & Freiman, 2016).

In any event, students needed to try different kinds of smaller objects in order to achieve their primary object, to drive along a circle with a radius of 1 m. First, the students used the circle circumference formula as a tool to determine the length of the route that the robot had to drive. However, the students did not realize that they made a mistake with the circle circumference formula, even though they had a short conversation about the value of the circle circumference.

> Lucas wrote on the whiteboard $1 \times 3.1415 = 3.1415$ and stated: *Because it is how many meters it has to drive.*
>
> Jacob was a bit skeptical with this: *Does it have to drive that many?*

Lucas: *Three meters. Yes, because we do have a radius of one and that is why it has to drive three meters, point one four or something like that.*

Because of Jacob's questioning, the students' focus was on the mathematical tool during this conversation. That state was only temporary because, after this conversation, the students just took the value of the circle circumference as a tool to use and they did not question its validity any further. Thus, the object remained, and mathematics worked as a tool to mediate the activity, even though the focus was temporarily on the tool.

Second, the students used the ratio of the circle circumference to the robot's wheel circumference to determine how many rotations the robot wheels had to rotate. They knew from earlier sessions that the robot's wheel circumference was approximately 17 cm. With the ratio 314 cm:17 cm, they used the calculator to determine that the robot wheels had to rotate approximately 18.5 times. This calculation was the result of common reasoning. Both of the students suggested different kinds of relations to determine the number of rotations required. Through common reasoning, they obtained the correct answer.

Third, the students used proportions to determine how to program the robot to make the turn with a proper angle. With Lego Mindstorm robots, it is possible to program turning on a scale of 1–100. The students began by determining what the scale 1–100 means in relation to the turning angle of the robot. Jacob determined that the value of 100 must mean the entire turn (360°). Students used proportions to determine how many degrees the robot turns with the value one.

After a short discussion, Jacob concluded: *50 is 180 degrees. And then, 25 is 90 degrees.*

The discussion of proportions continued later. Meanwhile, they determined how many degrees the robot had to turn during one wheel rotation.

Lucas: *The robot has to spin 360 degrees, so it will be 360 divided by 18.5.*

Lucas calculated 360:18.5 using the computer and obtained an answer of approximately 19.5. Then, the students continued using proportions to determine what value they had to use to program the robot to turn with the correct angle.

Lucas: *Because, we only have up to 100, we have to divide 360 by 100, which is 3.6. Isn't it?*

Jacob: *Yes, 3.6.*

Lucas: *Yes, I hope that is correct*. *3.6, ok, so that means that 1 is the same as 3.6 and we have to divide 19.5 by 3.6.*

After a discussion, Lucas used a calculator to obtain the answer 5.5.

Jacob: *Let's try this out.*

With the help of these different proportions, the students found the correct values to use in their program to make the robot turn in the desired angle. According to our analysis of Jacob's last statement, after the students obtained the required values with the help of mathematical tools, they were ready to test the values in their program, and their focus shifted again from the tools to the object.

In summary, in each of these respects, mathematics was used as a tool for reaching the object, to make a robot drive along a circle with a radius of 1 m. More specifically, the circle circumference formula was used to determine the distance the robot had to drive, and proportions were used to determine how much the robot had to turn. Even though the students' attention was temporarily on the tools, these mathematical tools still remained as tools and not as objects of the activity. After the students obtained the required answers using their mathematical tools, they were willing to use their answers to make the robot drive along a circle with a radius of 1 m, the object of the students' activity. Students simply fed the required values into their program and tested it. The students' focus then was on the testing of the program and on the robot, no longer on the mathematics.

The discussed mathematical tools, namely, circle geometry and proportions, can both be connected with the mathematics curriculum. According to earlier studies, the connections between robot-based activities and curriculum have been unclear (Alimisis, 2013; Benitti & Spolaôr, 2017; Karim, 2015). As the students did not receive any external help, such as information or advice from a teacher, a book, or the Internet regarding mathematical tools, the students used the mathematics that they already knew. However, the use of mathematical tools occurred through collaboration between the students. Both of the students contributed when they were designing the use of mathematical tools or when they were using the mathematical tools. The students alternated between different roles, alternately coming up with different ideas, conducting different calculations using the computer, or writing their ideas and calculations on the whiteboard. The students used the mathematics that they already knew, but their knowledge was strengthened through collaboration, and they were able to apply their knowledge to mediate the activity.

In any event, the use of mathematical tools did not always follow the formal mathematical rules, for example, when the students wrote on the whiteboard 360 = 100, which means that the entire turn of 360° is equal to the value 100 in the program (see Picture 1). The use of free rules in robot-based activities makes the use of mathematics more informal, and thus, students forget the use of formal rules in mathematics. This is in alignment with the argument of Barak and Assal (2018) regarding the challenges of teaching and learning formal mathematics through informal activities with robots. According to Barak and Assal (2018), the informal nature of robot-based activities makes the formal use of mathematics or other science, technology, engineering, and mathematics (STEM) subjects challenging. Furthermore, during free activities with robots, the teacher refrains from interfering in the informal use of STEM subjects.

Phase 3: Object development

The use of mathematical tools depended on the object development. As discussed earlier, the common object, to program the robot to drive along a circle with a radius of 1 m, induced students to design the use of mathematical tools. The further development of the object is discussed in the following.

At some point, when the students were working with the mathematical tools, the teacher realized that the students made an error with the circle circumference formula. The teacher attempted to encourage the students to pay attention to their mistake with the mathematical tool when the students were conducting their reasoning to determine the values required to program the robot.

> The teacher: *I am just wondering, where, how, you got 314.15 centimeters from?*
>
> Jacob looked skeptically at the teacher: *How? What?*
>
> Both of the students looked at the whiteboard, and Lucas gave the answer: *Oh, yes. Because we had to multiply one meter by pi and we had 17 centimeters with one wheel rotation, so we transformed it to centimeters.*
>
> Jacob continued: *So, we ended up with that it has to drive 314 centimeters*.
>
> The students continued working, but the teacher did not give up: *How did you determine to multiply the radius by pi?*

Lucas looked at the teacher skeptically and then looked at Jacob. He laughed a bit uncomfortably: *I do not know how, I do not remember it now.*

The students continued working without paying any more attention to the teacher's question. They did not want to pay any attention to their mathematical tool in that phase because they were committed to their object and they did not see any point to it. According to our analysis regarding the students' gestures, such as skeptical looks and uncomfortable laughter, the students did not have any idea what the teacher was talking about, and they did not care because they did not want to pay attention to the teacher's question or the mathematics behind the question. Both the mathematics and the teacher were excluded from the object of the students. They just wanted to continue towards their object to drive a circle with a radius of 1 m, the driving force in this phase. Thus, the teacher could not change the students' focus or object with his questions in this phase (e.g., Engeström, 1987).

In any event, after the students had input the needed values, obtained using the mathematical tools, into the program, they tested it, and the robot drove only a half circle. To determine what went wrong, both of the students concentrated and shifted their focus temporarily to the mathematics again. Lucas went through their calculations on the whiteboard while Jacob sat on the computer.

After some thinking, Jacob suggested: *We try 37.*

Lucas: *Why?*

Jacob: *It is double, because the robot drove only half way.*

Lucas accepted this solution with a smile: *Good plan, we say.*

However, the focus on mathematical tools was only temporary because, after Jacob's suggestion, the students just doubled their answer, used it on their program, and succeeded in their task. Lucas did not even realize why Jacob wanted to double the answer, but his smile showed his satisfaction with Jacob's suggestion. The students just wanted to reach their object, and they were not further interested in the source of the error or the reason why they had to double their answer. They were only interested in reaching their object, to make the robot drive along a circle with a radius of 1 m.

Finally, the students succeeded in reaching their object, their goal, and they were satisfied with the result. Next, there was space for negotiation regarding a new object, a new motive or goal.

After the students succeeded, the teacher continued: *But now you have to determine why*.

Jacob: *We just doubled it.*

The teacher: *Yes, why did you double it?*

Jacob: *Because we saw that it drove only half way*.

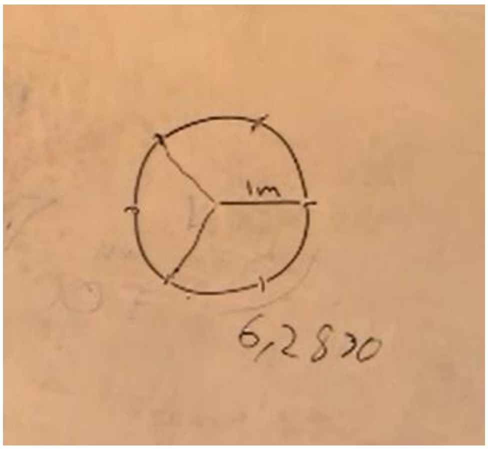
The students were excited about their success, and they were not interested in thinking about it further.

Jacob: *But we managed to do it before the end of this session*.

The teacher: *Yes, that is impressive. But tell me what you calculated*.

After a couple of jokes, Lucas smiled and accepted the teacher's suggestion to explain why doubling the answer worked. The students and the teacher discussed circle geometry on the whiteboard. Picture 2 is the whiteboard view after the discussion.

Picture 2. Whiteboard view after the students' and teacher's discussion of circle geometry



The teacher drew a circle on the whiteboard and pointed to its radius: *So, this is one meter*.

Then, he pointed to one part of the circle circumference: *So, we estimate that this is about one meter, or is it? Is it about one meter?*

Lucas: *1/3 is one meter.*

The teacher pointed to about one third of the circle circumference: *1/3 from here. So, from here to there? Should we call this one meter, then? Can you draw the arc then, one meter? Say we start here. How far away is about as far as this, then?*

Lucas divided the circle circumference into three different arcs. The teacher pointed to the radius of the circle: *As far as here, about?*

After some hesitation from the students, the teacher pointed to the radius again: *Can you show with your fingers how long you think this is then? How long?*

This discussion continued and Jacob divided the circle circumference into arcs of approximately the length of the radius. Students determined that the circle is approximately 6 m in length.

The teacher continued: *Yes, how do we find the circumference of a circle then?*

Lucas: *You have to take it twice as much as it is and then…Double radius multiplied by pi.*

This discussion ended with Lucas's statement: *Now we know, why we had to double it.*

Finally, the teacher moved the students deeper into circle geometry, with a new driving force for the conversation being to determine why they had to double their answer. The students explained their calculations to the teacher, with mathematics becoming the object of the students' activity. The focus was not just temporarily on mathematics; mathematics was the drive and direction in the activity. The students concentrated only on mathematics, because they had reached their original object to drive a circle and they did not have to get back to the original object anymore. Mathematics was not just a tool anymore, where the focus is only temporary. The focus was on the mathematics until students reached their new object, to find out why they had to double their answer.

This change in the students' object was a result of the teacher's steadfast negotiation with the students at the proper moment during the activity development, as the collective object can change during the activity development as a result of the manipulations of the activity participants (e.g., Kuutti, 1996).

The object change was an interesting turning point in the students' learning process. First, without the object change, the students would have been satisfied with reaching their original object. Thus, the students would not have discovered the source of the error, which has been seen in earlier studies as an obstacle to acquiring greater understanding in mathematics (Savard & Freiman, 2016).

Second, before the object change, mathematics was used as a tool, and the students' attention was not on it. When mathematics was used as a tool, students used their mathematical knowledge in action informally. In any event, when mathematics became the object of the activity, the students paid attention to it and obtained new knowledge with it, giving the teacher an opportunity to teach formal mathematics. That mathematics became the object of the activity does not mean that it displaced technology and the robot. On the contrary, the robot and mathematics merged into a hybrid and expanded object.

Third, the object change was dependent on the role of the teacher during activity development. When the teacher realized that the students erred with the circle circumference formula, he could have just corrected the mistake by mentioning it to enable the students to double their answer. That advice would have stopped the development towards object change, and mathematics would have remained just some informal tool in the activity. However, the teacher did not do that but decided to follow the students work and, when possible, to ask relevant questions and negotiate with the students without providing any ready tools to use. The object change was not externally provided but was a result of the process of student activity. The teacher was a mediator in the process.

Phase 4: Expansion of the object

During Session 2, Oscar was again present, with Lucas and Jacob satisfied with their success last time and willing to apply their learning in a new situation. They wanted the robot to drive along a circle of a different size as a part of their new task.

> Lucas: *Did you delete the program, which made it drive a circle with a radius of one meter? It was a program with lots of mathematics in it.*
>
> Jacob smiled: *A lot of mathematics in it*.
>
> Lucas talked by emphasizing the word mathematics and enthusiastically waved his hands: *I have an idea. Now, we are going to do this with lots of mathematics, do you understand? ... Yes, we are going to make that big circle and we are going to use mathematics*.
>
> Lucas started to measure the diameter of the circle, which he had built with Lego bricks with the idea of programming a robot to drive around this circle as part of its pathway on the track that the students were planning to create. Jacob helped him: *How long a diameter does it have?*

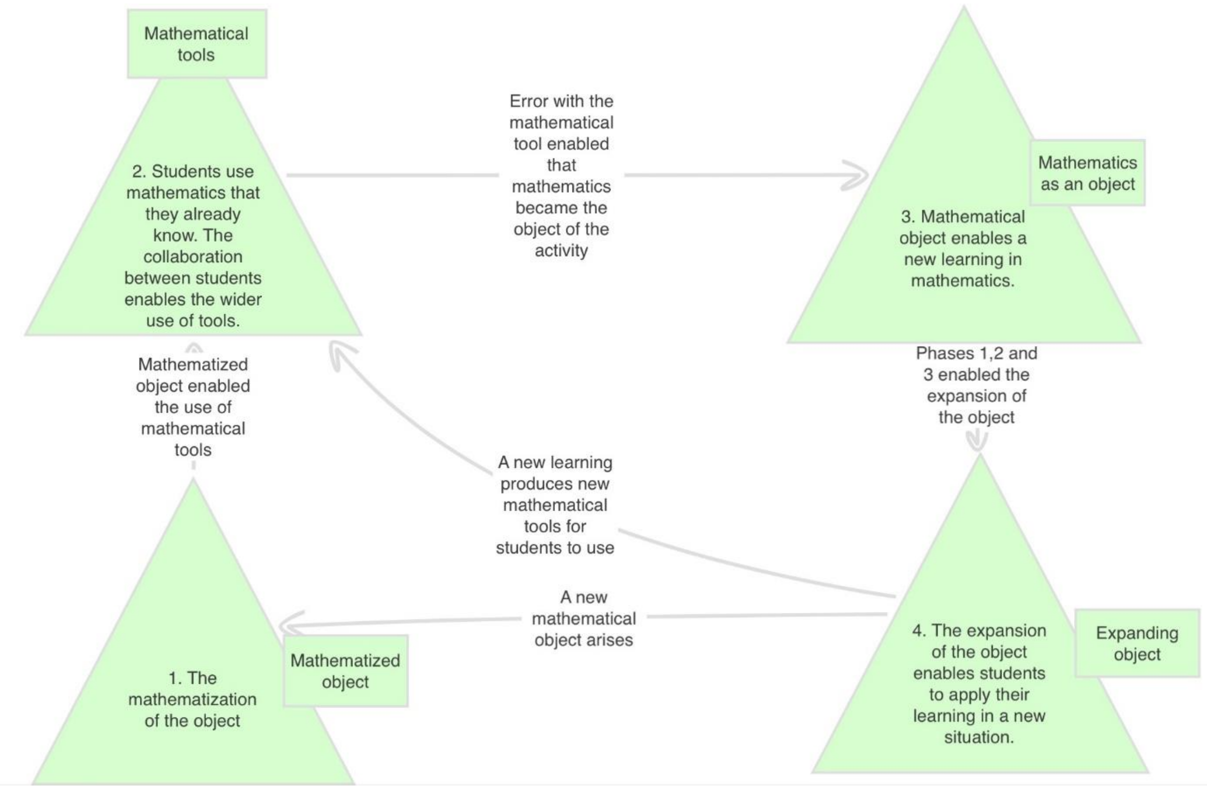
Lucas conducted the required calculations on the whiteboard for a new circle in the new situation. Mathematics was the driving force in this situation. The word mathematics had a positive tone in the conversation; Lucas was excited about mathematics. Lucas' excitement manifested in the way that he talked about mathematics and waved his hands enthusiastically. The word mathematics made Jacob smile, and mathematics was something to strive for. Important for the students was that the new object in a new task had mathematics that they learned last time in it. Because of the students' learning process and their satisfaction with their success from the last session, the students were willing and able to apply their learning in a new, wider situation. The mathematical object expanded (e.g., Engeström, 1987).

In summary, the use of mathematics and the expansion of the object consisted in specific turning points in the activity development (Figure 2). In the first phase (figure 2 and table 2), the students worked towards their object, which was mathematized. The students' original object was to drive in a circle, with the teacher suggesting the size of the circle. That is, the teacher refined the students' original object with mathematical details.

In the second phase (figure 2 and table 2), the mathematized object enabled the use of mathematical tools. At this point, the students used their mathematical knowledge to achieve their mathematized object. The students used the mathematics that they already knew by applying their knowledge in action, which was possible through collaboration. Thus, the use of mathematics was in relation with the students' collaboration. The collaboration between students developed the activity towards their common object.

In the third phase (figure 2 and table 2), the mathematical tool became the object of the activity after the students had reached their original object. Finally (phase 4 in figure 2 and table 2), the object was expanded when the students used their learning in a new wider context. The students created a new activity with a new mathematized object, which enabled the use of their new learning as a tool to mediate the new activity.

Fig. 2 Different turning points in the activity development towards the expansion of the object (Inspiration Maps).



None of these turning points alone would have enabled this kind of development. Thus, these different steps are strongly intertwined with each other. Furthermore, all of these steps stem from the students' first object, to drive a circle with a robot, which was the driving force during the entire process. These steps would not have been realized without the original object with the robot.

Conclusions and discussion

Our review of existing studies showed that the educational benefits of robotics in mathematics education are unclear, at least in part because they occur in complex environments involving digital tools, mathematical concepts and alternative pedagogies. In our research, we addressed the question of students' learning through analysis of the object development, not as changes in subjective knowledge, which quantitative studies on students' learning concentrate on.

Qualitative discussions regarding students' learning processes with robots indicated that the trial-and-error strategy for solving the problems functioned as an obstacle to finding the source of the error with the mathematical context (Savard & Freiman, 2016). By contrast, we introduced one case in which students avoided the trial-and-error strategy and used mathematical tools in robot-based activities. In the perspective of CHAT, the trial-and-error strategy differs from the activities of this study regarding the mathematical tools in use. In a trial-and-error strategy, students select the needed values randomly and mathematical tools are not used systematically. In this study, students did not select the values they needed in their programming randomly; they used mathematical tools systematically instead.

Furthermore, the activity developed towards expansion of the object through the error students made with the mathematical formula. Because the trial-and-error strategy was avoided, the students could detect the source of the error (cf. Savard & Freiman, 2016) and were enabled to create an expanded and hybrid object in which mathematics was merged with technology.

Barak and Assal (2018) argued that, even if problem-solving activities with robots provide rich learning experiences in mathematics, robotics cannot substitute for systematic mathematics teaching. We argue that, even if systematic formal mathematics teaching is not possible with robotics, at least in a traditional teacher-led manner, the teaching of formal mathematics is still possible as found also in Bartolini Bussi and Baccaglini-Frank (2015) with younger students. The younger students in Bartolini Bussi and Baccaglini-Frank (2015) deepened their understanding about rectangles in a practical context with robots. In this study, through the mistake that the students made with the circle circumference formula, the teacher took advantage of an opportunity for a thorough teaching session in circle geometry. A clear and practical connection with robots made the formal teaching session special and rich. Students learned how to use circle geometry in practice, their understanding about formal circle geometry deepened. Our finding related to the connections between formal mathematics and robot-based activities strengthens the idea of Bartolini Bussi and Baccaglini-Frank (2015) that activities with robots have the potential to open also other formal mathematical meanings for the students.

In any event, the opportunity for a rich learning session was not self-evident. The activity development described above was the result of particular incidents that are not directly generalizable. The students' learning could not have been predicted beforehand as their learning depended on the above-mentioned turning points in the activity development, which depended on choices and decisions that the students made. However, our point is that formal mathematics teaching is still possible with robots. The teacher cannot make a teaching plan that is as clear and as detailed as in traditional mathematics education, but curriculum connections can still be made in a more formal manner through robot-based activities (cf Alimisis, 2013; Benitti & Spolaôr, 2017; Karim et al., 2015).

We argue further that robot activity as analyzed in this case opens the possibility for curricular mathematics to be an integral part of the object of an activity in school. Mathematics is transformed from a means of assisting the joy, energy, and motivation of succeeding with the robot activity to a part of the motivational object itself. Thus, the activities with robots have the potential in mathematics education by providing a motivational environment for mathematics learning.

The limitation of this study is that the teacher of this study did not have a relevant programming background, and thus, the programming task assignments were not as advanced regarding programming. The activity development could have been even stronger with more advance programming tasks, developed by the teacher. This study has concentrated on the activity development in the beginning of robot integration. In further activity development, there is also a need to focus on the programming skills development, for diverse development of robot-based activities in classrooms. Thus, teachers' programming skills are what needs to be considered in mathematics teacher education and in teachers' further education. Anyhow, the role of the teacher in students learning processes with robots in the situation where the teacher does not have any programming background is discussed more detailed in our earlier article (Forsström, 2019).

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